frequency optimum isolator performance, for a frequency different from the given fixed frequency, shows that very large amplitude oscillations occur. It is necessary, therefore, to explicitly treat the variation of the free parameter within known or projected bounds.

The technique of treating the free parameter variation, by enforcing constraints and computing the cost function only at a finite grid on the free parameter domain appears to yield satisfactory results. There remains a need for research to develop a more efficient solution technique.

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Technical Comments

Comment on "Subsonic and Supersonic Boundary-Layer Flow Past a Wavy Wall"

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Introduction

RECENTLY Inger and Williams[1] investigated both theoretically and experimentally the problem of turbulent boundary-layer flow over a wave-shaped wall in the Mach number range of 0.8 to 1.8 at unit Reynolds number on the order of 10^6 per in. An essential feature of the inviscid part of Inger and Williams' study was the development of a "top-down" integration scheme whereby the two-point boundary value problem was converted into an "initial value" problem. The purpose of this Note is to point out that their calculation is only valid near the wall, and to show the appropriate process of converting into an initial value problem.

Analysis

The differential equation pertaining to the problem of a two-dimensional, steady compressible flow past a sinusoidal wavy wall of small amplitude has been derived before,[1,2] and it is in the form

$$\frac{d^2\tilde{p}}{dy^2} - 2\frac{dM/dy}{M}\frac{d\tilde{p}}{dy} + \alpha^2(M^2-1)\tilde{p} = 0 \tag{1}$$

where $\tilde{p}$ is the complex variable of pressure perturbation, which is defined as $p = P_e + \tilde{p}(y)\,e^{i\alpha x}$, with $\alpha = 2\pi/\lambda$ the wave number, and $M(y)$ the meanflow local Mach number. The boundary conditions are[1,3]

$$d\tilde{p}/dy = -i(M_e{}^2-1)^{1/2}\alpha\tilde{p} \qquad \text{at} \qquad y = \delta \tag{2}$$

and

$$d\tilde{p}/dy = -\varepsilon\alpha^2\rho u^2 \qquad \text{at} \qquad y = y_f \tag{3}$$

where M_e is the outer-edge freestream Mach number, and y_f is a "cut-off" distance used in place of $y = 0$, since the solution is singular at $y = 0$ where $M \to 0$. The factor ε is the wave amplitude, ρ the mean flow density, and u the velocity. The outer-edge boundary condition, Eq. (2), implies that the corresponding disturbed pressure field behaves either like simple waves ($M_e > 1$) or exponentially decaying signals ($M_e < 1$). The inner boundary condition, Eq. (3), represents a kinematic tangency condition.

This is a two-point boundary value problem which can be solved by conventional "shooting" methods. However, instead of using the condition at the inner boundary, Eq. (3), Inger and Williams adopted the solution for uniform potential flow past a wavy wall as a second outer-edge boundary condition; namely, $\tilde{p} = \tilde{p}_{\text{potential}}$ at $y = \delta$. This converts the study into an initial value problem and is justified only if $\tilde{p}$ is properly scaled after the calculations.

To illustrate the appropriate conversion process, let $Q(y)$ be the solution of Eq. (1), which is defined by the boundary conditions

$$dQ/dy = -i(M_e{}^2-1)^{1/2}\alpha Q \qquad \text{at} \qquad y = \delta \tag{4}$$

and

$$Q = Q_o \qquad \text{at} \qquad y = \delta \tag{5}$$

It is easy to show, by virtue of the linearity of Eq. (1) and conditions of Eqs. (2) and (4), that $\tilde{p}$ is a simple multiple of the solution $Q(y)$. By condition of Eq. (3), the required multiple is

$$\tilde{p}(y) = -\left.\frac{\varepsilon\alpha^2\rho u^2}{dQ/dy}\right|_{y=y_f} Q(y) \tag{6}$$

Note that, in general, Q_o can be any number, complex or real, and Eq. (6) will give the proper scaling that converts $Q(y)$ into $\tilde{p}(y)$, the expected physical answer. In our example, we set Q_o equal to the potential solution at $y = \delta$.

In the present investigation, the Adams-Bashforth-Moulton predictor-corrector scheme[4] is employed to integrate the differential equation across the boundary layer. The mesh size is automatically adjustable to stay within the accuracy criterion

$$\varepsilon_1 < \left|\frac{\tilde{p}_{\text{predictor}} - \tilde{p}_{\text{corrector}}}{\tilde{p}_{\text{corrector}}}\right| < \varepsilon_2$$

where ε_1 and ε_2 are assigned small quantities such as $\varepsilon_1 = 10^{-5}$ and $\varepsilon_2 = 10^{-2}$.

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Index categories: Boundary Layers and Convective Heat Transfer — Turbulent; Subsonic and Transonic Flow.

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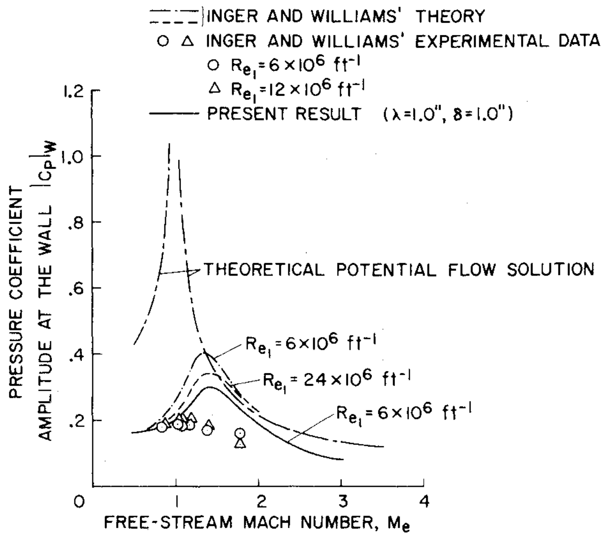

Fig. 1 Amplitude of pressure coefficient at the wall for different free-stream Mach numbers.

For a simple study, the gas is assumed to be perfect, the turbulent boundary layer is approximated by a $\frac{1}{7}$th-power velocity law, and the temperature is obtained from the Crocco's relation for an adiabatic wall

$$\frac{T}{T_{aw}} = 1 - r\frac{\gamma-1}{2} M_e^2 \frac{T_e}{T_{aw}} \left(\frac{u}{u_e}\right)^2$$

where T_{aw} is the adiabatic wall temperature, r is the recovery factor, u_e is the freestream velocity, and T_e the temperature. The mean-flow density $\rho(y)$ and Mach number $M(y)$ across the boundary layer are then calculated. The calculations are started by using Eq. (4) and the potential solution for Q_o at $y = \delta$. Then the multiple constant is determined at the "cut-off" distance which is evaluated from $y_f = 0.776(\mu_w^2/\rho_w \tau_w \alpha)^{1/3}$ where τ_w is the shear stress at the wall, and μ_w and ρ_w are the viscosity and density evaluated at wall temperature. The results were scaled accordingly.

Results

Computed results were obtained for pressure coefficient distribution ($\tilde{C}_p$) at both subsonic and supersonic Mach numbers. Figs. 1 and 2 show the resulting amplitude ratio and phase shift of pressure coefficient at the wall as functions of freestream Mach number. These results are qualitatively in good agreement with the theoretical and experimental results of Inger and Williams. However, it is recognized that, for better comparison,

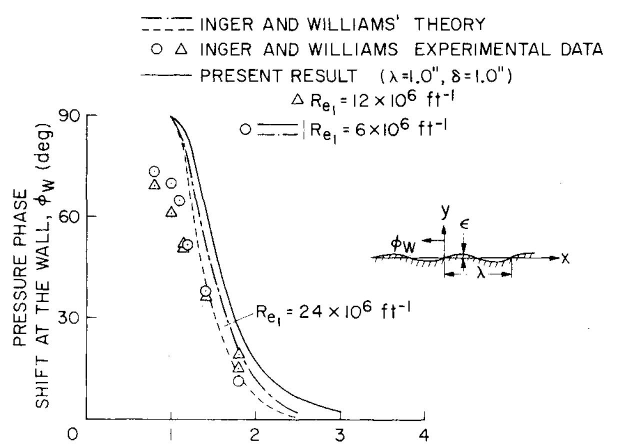

Fig. 2 Pressure phase shift at the wall for different freestream Mach numbers.

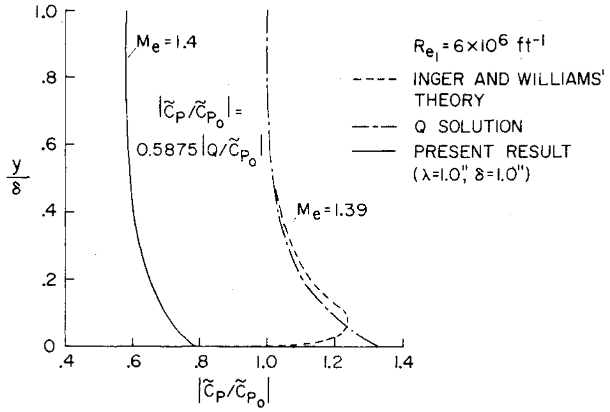

Fig. 3 Pressure amplitude ratio across the boundary layer.

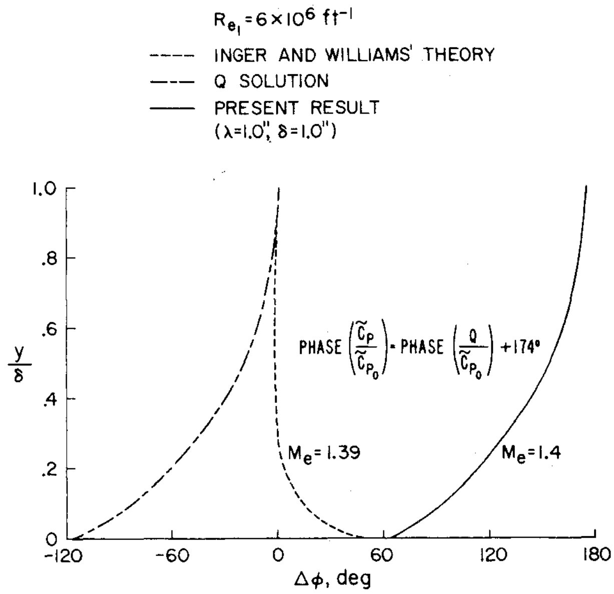

Fig. 4 Phase shift across the boundary layer ($\Delta\phi = \phi - \phi_{\text{potential}}$).

a calculation for the local mean-flow Mach number and boundary-layer thickness (more precisely speaking, the ratio of boundary-layer thickness to wave length) is necessary.

The present predictions of pressure amplitude and phase shift across the boundary layer are shown in Figs. 3 and 4 for $M_e = 1.4$. We found that the multiple for $\tilde{C}_p$ and Q, in which $Q = \tilde{C}_{p_o}$ at $y = \delta$, is

$$\tilde{C}_p(y) = 0.5875\, e^{i\,174^\circ}\, Q(y)$$

As expected, $\tilde{C}_p$ is quite different from $\tilde{C}_{p_o}$ at $y = \delta$. The comparisons shown in these two figures indicate that, except very near the wall, the numerical results reported by Inger and Williams are drastically different from the expected physical answer.

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